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# 액정 p-Azoxyanisole 의 열역학적 성질에 대한 이론적인 예측

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# Theoretical Prediction of the Thermodynamic Properties of Liquid-Crystalline p-Azoxyanisole

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요 약. 액체에 대한 significant structure theory 와 상의 전이에 대한 Bragg-Williams 근사를 사용하여 액정 화합물인 p-azoxyanisole의 열역학적 성질을 nematic phase 와 isotropic phase의 온도범위에 걸쳐 계산하였다. Isotropic phase는 일반적인 액체로 보았으며 nematic phase는 액체적인 성질의에도 분자쌍극자의 배열에 의한 영향도 고려하였다. p-Azoxyanisole의 액체적인 성질은 significant structure theory로 기술하였으며 분자쌍극자 배열에 의한 영향은 Bragg-Williams 근사로써 고려하였다. 부피, 증기압, 정압비열, 열팽창계수, nematic-isotropic 전이점에서의 엔트로피 엔탈피 변화, 절대엔트로피, Helmholtz free energy 등을 계산하여 실험치와 비교하였다.

ABSTRACT. The significant structure theory of liquids and the Bragg-Williams approximation of phase transition theory have been applied to the calculation of the thermodynamic properties of p-azoxyanisole which exhibits a liquid crystal phase of the nematic type. The isotropic phase was treated as a normal liquid; and for the nematic phase, in addition to its liquidity, the effect due to the arrangement of molecular-dipoles was considered. The liquidity of the p-azoxyanisole was described by the significant structure theory of liquids, and the Bragg-Williams approximation was used to consider the effect due to the arrangement of molecular-dipoles. The molar volume, vapor pressure, heat capacity at constant pressure, thermal expansion coefficient, compressibility, entropy and enthalpy change at the nematic-isotropic phase transition point, absolute entropy, and absolute Helmholtz free energy were calculated over the temperature range of the nematic and isotropic phases. The calculated results of the thermodynamic properties were compared with the experimental data.

#### 1. INTRODUCTION

Liquid crystals have been an object of scientific

curiosity because of its wide applicability<sup>1~4</sup> and its abnormal thermodynamic properties. <sup>5~9</sup> In line with the rapid increase in interest for liquid

crystals, a number of experimental studies about the structure and the properties of liquid crystals have been carried out extensively. But according to our historical survey about the theoretical works,  $^{7\sim13}$  it has been left as an important problem to predict theoretically the background quantities of the thermodynamic properties (Fig. 1), the surface tension, and the transport properties of liquid crystals.

Maier and Saupe<sup>7</sup> and Alben<sup>8</sup> applied a general theory of phase transition described by a longrange order parameter to nematic p-azoxyanisole. Chandrasekhar et al. 10 obtained the contribution of the disorder part using the extended theory of the two-lattice model of Lennard-Jones and Devonshire. Imura and Okano<sup>12</sup> calculated the excess quantity of the heat capacity and the thermal expansion coefficient due to the orderparameter fluctuation by using the continuum fluctuation theory of de Gennes. Ma and Eyring<sup>13</sup> extended the significant structure theory of liquids to the mesophase system. In their calculations, a few properties were evaluated

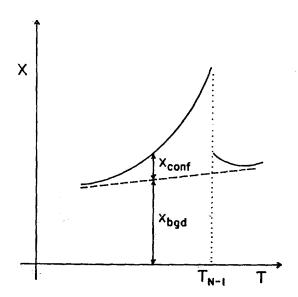


Fig. 1. Schematic representation of the background  $(X_{\text{bgd}})$  and configurational  $(X_{\text{conf}})$  contributions to the thermodynamic properties X of p-azoxyanisole.

without introducing order-disorder concepts.

The p-azoxyanisole system was chosen in this research because better and sufficient experimental results can be found in the literature. The molecule of p-azoxyanisole (PAA; 4, 4'-dimethoxyazoxy-benzene) has a permanent dipole of 2. 28 Debye unit<sup>14</sup> and the molecular weight of 258. 28. The molecule of crystalline p-azoxyanisole consists in the  $C_{2h}^5$  group with four molecules per unit cell. 15 The molecules are fixed in position with the long axes aligned approximately perpendicular to the (100) plane: they form an imbricated structure with the benzene rings of the two neighboring molecules either facing or perpendicular to each other. The CH3O groups should then be in close contact with the N2O groups. In the nematic phase, the molecules are no longer rigidly fixed in position. They are still well aligned, but they can rotate more or less freely about their own long axes. 16, 17 The nematic range of p-azoxyanisole extends from about 391.4 to 408°K. 18~22 In the isotropic liquid phase, all long-range ordering breaks down including the ordering in the molecular alignment.

In this paper, using the basic concepts of the significant structure theory of liquids and the Bragg-Williams approximation of phase transition, the thermodynamic properties of *p*-azoxyanisole are evaluated and the calculated results are compared with the observed values.

## 2. THEORY

Partition Function of Nematic p-Azoxyanisole. According to the density measurement of p-azoxyanisole<sup>7, 20, 22, 23</sup> the density change of the nematic-isotropic transition is about 0.36 % whereas the density change at the crystal-nematic phase transition is in the range of  $7 \sim 11$  %. This indicates that the nematic phase has many fluidized vacancies and also that the theory of

liquids such as the significant structure theory of liquids can be applied to nematic p-azoxyanisole to explain the liquidity of liquid crystal. In spite of increasing the fluidized vacancies, the molecules of nematic p-azoxyanisole are still well aligned along their long axes and have long-range ordering. Therefore, for sufficient understanding of the system of liquid crystal, it is necessary to have other partition function including the term of long-range ordering. So the partition function of nematic p-azoxyanisole can be written as

$$f_{L \cdot C \cdot} = f_{\text{bgd}} \cdot f_{\text{conf}} \tag{1}$$

where  $f_{L\cdot C}$  is the partition function of nematic p-azoxyanisole,  $f_{bgd}$  is the partition function which is introduced to take into account the aspects of the liquidity of nematic p-azoxyanisole, and  $f_{conf}$  is the partition function including the long-range ordering of nematic p-azoxyanisole.

As illustrated in Fig. 1, it is reasonable to think that any thermodynamic properties of nematic p-azoxyanisole, X, is given as the sum of two terms,  $X_{\rm bgd}$  and  $X_{\rm conf}$ . The term  $X_{\rm bgd}$ , called background quantity, is the contribution due to the background partition function,  $f_{\rm bgd}$ , and the configurational quantity,  $X_{\rm conf}$ , is the contribution due to the configurational partition function,  $f_{\rm conf}$ .

Using the basic concepts of the significant structure theory of liquids, <sup>24, 26</sup> the background partition function of nematic *p*-azoxyanisole can be written as:

$$\begin{split} f_{\text{bgd}} &= \left[ \frac{e^{E_s/RT}}{\cdot (1 - e^{-\theta/T})^5} \left\{ 1 + n \left( \frac{V - V_s}{V_s} \right) \right. \right. \\ &\left. \cdot \exp \left( - \frac{aE_s V_s}{RT \left( V - V_s \right)} \right) \right\} \\ &\left. \cdot \frac{2\pi (2\pi I_C kT)^{1/2}}{\sigma h} \cdot \frac{kT}{B} \sinh \left( \frac{B}{kT} \right) \right. \\ &\left. \cdot \prod_{i=1}^{93} (1 - e^{-h\nu_i/kT})^{-1} \right\}^{NV_s/V} \end{split}$$

$$\times \left[ \frac{(2\pi mkT)^{3/2}}{h^3} \cdot \frac{eV}{N} \right] \\ \cdot \frac{8\pi^2 (8\pi^3 I_A I_B I_C)^{1/2} (kT)^{3/2}}{\sigma h^3} \\ \cdot \frac{kT}{B} \sinh \left( \frac{B}{kT} \right) \\ \cdot \prod_{i=1}^{93} (1 - e^{-h\nu_i/kT})^{-1} \right]^{N-V-V_5}$$
(2)

Here, m is the mass of a molecule; R is the gas constant; h is Plank's constant; k is Boltzmann's constant; N is Avogadro's number; e is the base of natural logarithm;  $\sigma$  is the symmetry number; a and n are the dimensionless parameters; B is the interaction parameter;  $I_A$ ,  $I_B$  and  $I_C$  are the principal moments of inertia;  $E_s$  is the heat of sublimation;  $\theta$  is the Einstein characteristic temperature;  $\nu_i$  is the vibrational frequency;  $V_s$  is the molar volume of the solid like melecule at the phase transition point; and V is the molar volume of p-azoxyanisole at temperature T in Kelvin degree. In the case of p-azoxyanisole,  $\sigma$  is unity.

The background partition function given in Eq. (2) was formulated readily as follows. nematic phase, p-azoxyanisole molecule rotates one-dimensionally about its long-axis. But the molecules have permanent dipoles and the dipoles of p-azoxyanisole are well aligned in the z axis. So in a macroscopic view, the molecule looks like as if it were in a field of potential  $V(=-B\cos\theta)$  where  $\theta$  is the angle between the z axis and the molecular dipole. This consideration was introduced to the one-dimensional rotational partition function of the solid-like molecule and to the three-dimensional rotational partition function of the gas-like molecule in the form of  $\left(\frac{kT}{B}\right) \cdot \sinh\left(\frac{B}{kT}\right)$ . Furthermore, the entropy change of the solid-nematic phase transition is about 18 e. u. 16, 18, 21 Hence, the degrees of freedom of Einstein oscillators were taken as five.

Nematic p-azoxyanisole obtained from crystal-

(6)

line p-azoxyanisole by heating has well aligned molecular arrangement. The order of the arrangement is broken down gradually as the temperature is raised and at the nematic-isotropic transition point the complete disordered state appears. This order-disorder phenomenon can be explained quantitatively by the theory of phase transition such as the Bragg-Williams approximation. 28, 29

Assuming that the dipole of a p-azoxyanisole molecule has only two states of up and down directions, and that all short range order parameters are the same, and neglecting the local statistical fluctuation, the application of the Bragg-Williams approximation to the evaluation of the configurational partition function is straightforward. Then the configurational partition function of nematic p-azoxyanisole for a given Q can be given as following equations.

$$f_{\text{conf}} = W(Q) \cdot \exp(-E(Q)/kT)$$
(3)  

$$W(Q) = {}_{N}C_{Na} \cdot {}_{N}C_{Nb}$$

$$= \left(\frac{N!}{(N - \frac{1 + Q}{2})! \cdot (N - \frac{1 - Q}{2})!}\right)^{2}$$
(4)  

$$E(Q) = N_{a}V_{a} + N_{b}V_{b}$$

$$= N - \frac{1 + Q}{2}V_{a} + N - \frac{1 - Q}{2}V_{b}$$

$$= \frac{N}{2}(U_{0} - V_{0}Q^{2})$$
(5)  

$$Q = \frac{N_{a} - N_{b}}{N_{c} + N_{b}}$$
(6)

Here Q is the long range order parameter;  $N_a$ N<sub>b</sub> denote the number of dipoles in the updirection and down-direction, respectively; W (Q) is the number of possible ways obtaining the  $N_a \cdot N_b$  state of the dipoles; E(Q) is the configurational internal energy of the system;  $V_a$  and  $V_b$  are the potential energy of the up and down directional dipoles, respectively; and  $U_0$  is the energy of the complete disordered state Q=0. Equations (3) to (6) were obtained by assuming that there are 2 N sites for N molecules and that the relation  $V_b - V_a = V_0 Q$  holds where  $V_0$  is a proportionality constant.

Partition Function of Isotropic p-Azoxyanisole. In the isotropic phase of p-azoxyanisole, the ordering in the molecular alignment and all long-range orderings are completely broken down. So there is no contribution from the configurational partition function,  $f_{conf}$ , and there is no field interaction effect. Thus the partition function of isotropic liquid of p-azoxyanisole,  $f_{I-L}$ , is given by Eq. (7).

$$f_{I \cdot L \cdot} = f'_{\text{bgd}} = \left[ \frac{e^{E_s / RT}}{(1 - e^{-\theta / T})^5} \left\{ 1 + n \frac{V - V_s}{V_s} \right\} \right] \cdot \exp\left( -\frac{aE_s V_s}{RT (V - V_s)} \right) \cdot \frac{2\pi (2\pi I_C kT)^{1/2}}{h} \cdot \prod_{i=1}^{93} (1 - e^{-h\nu_i / kT})^{-1} \right]^{NV_s / V} \times \left[ \frac{(2\pi m kT)^{3/2}}{h^3} \cdot \frac{eV}{N} \right] \cdot \frac{8\pi^2 (8\pi^3 I_A I_B I_C)^{1/2} (kT)^{3/2}}{h^3} \cdot \prod_{i=1}^{93} (1 - e^{-h\nu_i / kT})^{-1} \right]^{N(V - V_s) / V}$$

$$(7)$$

All notations in Eq. (7) have the same meaning as in Eq. (2).

### 3. CALCULATIONS

All thermodynamic properties of p-azoxyanisole were calculated over the temperature range of anisotropic and isotropic phases using NOVA 840 computer. The partition function is closely related to the Helmholtz free energy, A, such as

$$A = -kT \ln f \tag{8}$$

The thermodynamic properties of p-azoxyanisole can be deduced with the use of Eq. (8).

In Table 1 and 2, the parametric values and other input values for p-azoxyanisole, which are used in the calculation of the background partition function, are given. The parametric values

Table 1. Parametric values for p-azoxyanisole used in the calculation of the background partition function,

Phase of PAA	$E_s$ (cal/mole)	θ (°K)	V <sub>s</sub> (cc/mole)	n	a×10 <sup>5</sup>	$B \times 10^{13}$ (ergs)
Namatic	22226. 2	78.40	218. 38	11.840	6. 6106	1. 4967
Isotropic	22226. 2	62. 17	222. 53	11. 831	7. 6162	_

Table 2. Vibrational frequencies and the moments of inertia used for p-azoxyanisole.

	Vibrational	frequencies (	cm <sup>-1</sup> ) <sup>17</sup>	
210	756	1114	1333	
234	797	1143	1370	
317	832	1157	1410	
360	848	1165	1425	
417	911	1171	1438	
474	943	1186	1454	
494	959	1203	1465	
536	1006	1220	1473	
611	1013	1246	1501	
629	1020	1252	1570	
670	1028	1261	1582	
717	1078	1276	1596	
725	1095	1301	1604	
753	1109	1319	1648	

Monents of inertia  $(g \cdot cm^2)^{32}$   $I_A = 7.50 \times 10^{-37}$   $I_B = 7.06 \times 10^{-37}$   $I_C = 4.71 \times 10^{-38}$ 

were determined by the Seoul technique;  $^{30,31}$  and a suitable value which gives good results in the thermodynamic calculations was selected for the interaction parameter, B. The order parameter, Q, which is used to calculate the configurational partition function of p-azoxyanisole is presented in Table~3. The equilibrium values of Q as a function of temperature were obtained by minimizing the Helmholtz free energy  $A_{\rm conf}$  with respect to Q, i. e.,  $(-kT \partial \ln f_{\rm conf}/\partial Q) = 0$ . This condition can be represented by the following equations:

$$Q = \tanh\left(\frac{V_0 Q}{2kT}\right) \tag{9}$$

Table 3. Order parameter Q for p-azoxyanisole used in the calculation of the configurational partition function.

T(°K)	Q
391. 4	1. 0000
393. 2	0. 9476
395. 2	0. 8853
397. 2	0. 8177
399. 2	0.7433
401. 2	0. 6598
403. 2	0. 5633
405.2	0.4454
407. 2	0. 2801
408. 5	0.0000

$$V_0 = 2kT_{N-I} \tag{10}$$

where  $T_{N-I}$  (=408.5 °K) is the nematic-isotropic transition temperature, and in deriving Eq. (9), the equation (12) which will be given shortly was utilized.

Molar Volume and Vapor Pressure. If the Helmholtz free energy is plotted as a function of volume at constant temperature, and a common tangent to the points corresponding to the liquid and vapor phases is drawn, the vapor pressure is given by the slope of the common tangent, and the abscissas of the two points indicate the respective volume of the liquid and vapor. The resulting values for the molar volume are given in *Table 4* and those for the vapor pressure are shown in *Fig. 2*. The calculated results give good agreement with the observed values.

Thermal Expansion Coefficient and Compressibility. The thermal expansion coefficient  $\alpha$  and compressibility  $\beta$  were calculated by using

Table 4. Molar volumes of p-azoxyanisole.

		•	
Т (°К)	$V_{ m obs}^{7}$ (cc/mole)	$V_{ m calc}$ (cc/mole)	% error
	Nemati	c phase	
391. 4	221.34	221. 34	0
393. 2	221.72	221.61	-0.05
395. 2	222.08	221. 92	-0.07
397. 2	222.44	222. 23	-0.09
399. 2	222. 81	222. 54	-0.12
401.2	<b>223. 1</b> 9	222. 85	-0.15
403. 2	<b>223.</b> 60	223. 17	-0.19
405. 2	224.05	223.48	-0.25
407.2	224. 51	223. 81	0. 31
408.5	224. 91	224.02	-0.40
	Isotropi	ic phase	
408.5	225.68	225.68	0
410.2	226.01	225.93	-0.04
411. 2	226. 19	226.08	-0.05
413. 2	226. 55	226. 37	-0.08
415. 2	226. 90	226.67	-0.10
417. 2	227. 24	226.96	-0.12

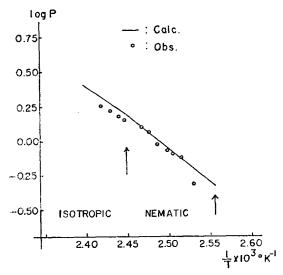


Fig. 2. Logarithm of p-azoxyanisole vapor pressures vs. reciprocal temperatures. Observed pressures in mmHg were obtained from ref. (33).

the partition function and the equations,

$$\alpha = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_P = -\frac{1}{V} \frac{(\partial P/\partial T)_V}{(\partial P/\partial V)_T}$$

Table 5. Thermal expansion coefficients of p-azoxy-

<i>T</i> (°K)	$lpha_{ m obs}{}^7 imes 10^4 \ (\ \  m K^{-1})$	$\substack{lpha_{ m calc} imes 10^4\ (^\circ { m K}^{-1})}$	% error					
	nematic phase							
391.4	7. 56	7. 24	-4.23					
393. 2	7.70	7.09	-7.92					
398. 7	8.31	••	• •					
399. 2	••	6. 99	••					
401. 2	8. 90	7. 02	-21.12					
403. 2	9. 33	7. 07	-24.22					
405. 2	9. 96	7. 13	-28.41					
407. 2	11.65	7. 20	<b>-38.2</b> 0					
408.5	14	7. 25	-48.21					
isotropic phase								
403.5	••	6.74	••					
410.2	8. 57	6.60	-22.99					
411.2	8. 13	6. 54	-19.56					
413. 2	7. 97	6.48	-18.70					
415.2	7. 63	6.46	-15.33					
417. 2	7.51	6.46	-13.98					

Other observed<sup>23</sup>  $\alpha$  of p-azoxyanisole.

Nematic region : 9.4×10-4°K-1 : 8.4×10<sup>-4</sup> °K<sup>-1</sup> Isotropic region

and

$$\beta \!=\! -\frac{1}{V} \left(\!\! \begin{array}{c} \! \partial V \\ \! \partial P \end{array}\!\! \right)_T \!\! = \!\! -\frac{1}{V (\partial P/\partial V)_T}$$

Comparisons between the calculated results with observed values of  $\alpha$  and  $\beta$  were given in Tables 5 and 6, respectively. There are two different experimental results about the thermal expansion coefficient: one is the λ-type temperature dependence by Maier and Saupe, 7 the other is the linear temperature dependence by Price and Wendorff. 23 Both experimental results were given in Table 5 for comparison with the calculated results. The experimental compressibility was obtained from a graph by Alben<sup>8</sup> whose result agreed with that of Hoyer and Nolle.9 Because the experimental transition temperature from nematic to isotropic phase is 405.2°K,

the compressibility was given as a function of  $(T-Tr)^{\circ}K$  where  $T_r(\equiv T_{N-I})$  is the nematicisotropic transition temperature.

Heat Capacity at Constant Pressure. The specific heat of p-azoxyanisole was also calculated by using the partition function and the equations

$$C_P = C_V + (TV\alpha^2)/\beta$$

and

$$C_V = (\partial E/\partial T)_V$$
.

The calculated results are summarized in Table 7. And the  $\lambda$ -type temperature dependence of p-azoxyanisole is illustrated in Fig. 3.

Entropy and Enthalpy Changes at the Nematic-Isotropic Transition The entropy, Helmholtz free energy, internal energy and enthalpy of p-azoxyanisole were calculated by using Eq. (8) in addition with the equations,

Table 6. Compressibilities of p-azoxyanisole.

<i>T−T<sub>r</sub></i> (°K)	$eta_{ m obs,~}^8 imes 10^{11} \  m (cm^2/dyne)$	$eta_{ m calc.}  imes 10^{11} \ ({ m cm}^2/{ m dyne})$	% error
-16	••	7. 29	
-14	••	7. 26	••
-12	••	7. 27	
-10	••	7. 32	••
-8	6.82	7.40	+8.50
-7	6. 98	7.47	$\pm 7.02$
-6	7. 14	7. 52	+5.3
-5	7. 34	7. 59	$\pm 3.41$
-4	7. 59	7.65	+0.79
-3	7. 86	7. 72	-1.78
-2	8. 27	7.80	-5.68
-1	8. 96	7. 88	-12.05
0	9.86	7. 95	-19.37
+0	••	7. 35	••
1	7. 25	7. 28	$\pm 0.41$
2	7. 11	7. 24	$\pm 1.83$
3⋅	7. 05	7. 23	+2.55
4	7. 00	7. 22	+3.14
5	7. 00	7. 22	+3.14
6	••	7. 23	••

$$S = -(\partial A/\partial T)_V$$

and

$$E=A+TS$$
.

In the case of nematic *p*-azoxyanisole, the configurational entropy and Helmholtz free energy are simply given as

$$S_{conf} = k \ln W(Q)$$

$$= Nk(2\ln 2 - (1+Q)\ln(1+Q)$$

$$- (1-Q)\ln(1-Q)$$
(11)

and

$$A_{\text{conf}} = \frac{N}{2} (U_0 - V_0 Q^2)$$

$$-NkT (2\ln 2 - (1+Q)\ln(1+Q)$$

$$-(1-Q)\ln(1-Q)). \tag{12}$$

 $U_0$  was obtained as  $1.5 \times 10^{-3}$  ergs by using the relation of  $\Delta H = T \Delta S$  at the namatic-isotropic

Table 7. Heat capacities of p-azoxyanisole at constant pressure.

	•		
<i>T</i> (°K)	$C_{ ho}(\mathrm{obs})^{5,6} \ (\mathrm{cal/g-deg})$	$C_{p}$ (calc) (cal/g-deg)	% error
	Nemati	ic phase	
391.4	0.4600	0.4420	-3,91
393. 2	0.4620	0. 4441	-3.87
395. 2		0.4471	**
397. 2	••	0. 4506	••
398.7	0. 4740	••	**
399. 2		0. 4543	••
401. 2	0.4842	0.4582	-5.37
403. 2	0.4970	0.4622	-7.00
404. 2	0. 5120	••	
405. 2		0.4663	••
407. 2	••	0.4703	••
408.5	••	0. 4732	• •
	Isotrop	ic phase	
408.5	••	0. 4475	
410. 2	0. 4850	0.4479	-7.65
411. 2	0.4680	0. 4483	-4.21
413. 2	0.4650	0. 4495	-3.33
415. 2	0.4658	0.4510	<b>−3. 18</b>
417. 2	0.4660	0.4528	-2.83

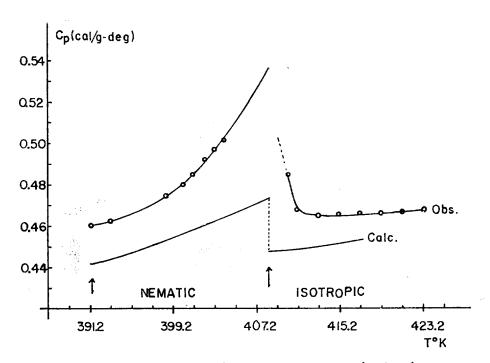


Fig. 3. Heat capacities of p-azoxyanisole at constant pressure as a function of temperature.

Table 8. Calculated absolute values of the entropy, Helmholtz free energy, internal energy, and enthalpy of p-azoxyanisole.

_								
-	T (°K)	S (cal/mole-°K)	A (cal/mole)	$E \approx H$ (cal/mole)				
_	Nematic phase							
	391.4	81.726	-38617.2	-6628.2				
	393. 2	82. 739	-38870.9	-6341.4				
	395. 2	83. 700	-39098.6	-6020.6				
	397.2	84. 619	-39308.9	-5697.1				
	399. 2	85. 507	-39506.8	-5372.9				
	401. 2	86. 359	-39698.5	-5052.1				
	403. 2	87. 227	-39888.1	-4718.0				
	405. 2	88.065	-40072.4	-4388.4				
	407. 2	88. 894	-40247.1	-4049.6				
	408.5	89. 429	-40369.8	-3838.4				
		Isotropi	ic phase					
	408.5	89. 758	-40370	-3704				
	410. 2	90. 237	-40523	-3508				
	411. 2	90. 519	-40613	-3392				
	413. 2	91. 080	-40795	-3161				
	415. 2	91. 643	-40978	-2928				
	417. 2	92. 204	-41162	-2695				

Table 9. Entropy and enthalpy changes of p-azoxy, anisole at the nematic-isotropic transition point.

	Calc.	Obs. 16, 18	% error
∆S(cal/mole-°K)	0. 329	0. 335	1. 79
$\Delta H({ m cal/mole})$	134. 4	137	1. 90

transition point (see Discussions). Absolute values of the entropy (S), Helmholtz free energy (A), internal energy (E), and enthalpy (H) of p-azoxyanisole are summarized in Table~8. The calculated values of  $\Delta S$  and  $\Delta H$  at the nematic-isotropic transition point are compared with the experimental values in Table~9. Since the vapor pressure of p-azoxyanisole are very small, E and E have almost same values.

# 4. DISCUSSIONS

Vibrational Normal Modes. For p-azoxy-

anisole, there are ninety-three vibrational normal modes. Although the observed lines in the Raman Spectra<sup>17</sup> are assumed to be normal modes, there are many missing normal terms. These terms were taken into account as follows:

$$\ln\left(f_{_{\mathrm{vib}}}^{M}\right) = -\sum_{i=1}^{m} \ln\left(1 - e^{-h\nu_{i}/kT}\right)$$
$$\simeq -m \cdot \ln\left(1 - e^{-h\nu/kT}\right)$$

where  $f_{\rm vib}^M$  is the vibrational partition function for the missing terms and m is the number of the missing normal modes. The essence of this consideration is that the missing vibrational frequencies are replaced by the parametric equivalent vibrational frequency. The calculated value of the parametric equivalent vibrational frequency which was determined from the experimental value of  $C_p$  at 391.4 °K is 674.4 cm<sup>-1</sup>.

Parameters used in Calculations. According to the complete Raman spectra<sup>17</sup> of p-azoxyanisole in the three phases, the intermolecular interaction energy in the substance is about  $10^{-13}$  ergs.

We consider why a nematic molecule of p-azoxvanisole has the potential energy of  $V=-B\cos\theta$ . The interaction energy of the dipole aligned in the Z direction with a dipole in any point is given by<sup>34</sup>

$$V = \frac{\mu_a \mu_b}{r_{ab}^3} \left[ -2 \cos \theta_a \cos \theta_b + \sin \theta_a \sin \theta_b \cos(\phi_b - \phi_a) \right]$$

where the subscript a indicates the dipole aligned in the z axis, and b expresses the dipole in a point in the space. Thus, the following equation results

$$V = \frac{\mu_a \,\mu_b}{r_{ab}^3} (-2\cos\theta_b) = -B\cos\theta$$

According to our results, the value of  $B(\approx 10^{-13})$  is the dipole-dipole interaction energy, and it is about equal to the interaction energy of

oscillators mentioned previously.

The heat of sublimation for benzene is about 11 kcal per mole. On the other side, the sum of the enthalpy changes at the phase transitions of p-azoxyanisole is in the range of  $21\sim26$  kcal/mole (cf.  $\Delta H$  (crystal $\rightarrow$ nematic)<sup>16,18</sup>=7.067 kcal/mole;  $\Delta H$  (nematic $\rightarrow$ isotropic)<sup>18</sup>=0.137 kcal/mole;  $\Delta H$ (isotropic $\rightarrow$ vapor)<sup>33</sup>=13.8 $\sim$ 18.8 kcal/mole). From these facts the value of  $E_s$ = 2226.2 cal per mole is justified.

The value of n is usually between 10 and 12. Thus, the values of n=11.840 and 11.831 are satisfactory. Although for normal simple liquids, the value of  $a \approx 0.0052^{25}$  is usually found, for water which has composite structure the value of a is  $1.653 \times 10^{-5}$ . 31 Thus, the values of a in Table 1 are acceptable.

Usually the  $\theta$  (Einstein) values for solid-like structures in liquids have been found smaller than the values for the solids. It is, of course, an oversimplification to take the librational frequency equal to the lattice vibrational frequency since the latter frequency is expected to be smaller than the former. According to the Raman spectrum for lattice vibrations of p-azoxyanisole single crystal, 32 the  $\theta$  value for lattice vibration is running from about 46 to 95 °K. The values of  $\theta$  in Table 1 lie within this range. The tendency that  $\theta$  for the nematic phase is higher than that for the isotropic phase is justified by the fact that the molecules in the nematic phase have less rotational freedom than those in the isotropic phase.

The values of  $V_s$  are to be compared with the experimental values of the molar volume of solid at the crystal-nematic transition point of about 199 to 220 cc/mole. <sup>7, 22, 23</sup>

Calculations of Configurational Partition Function. The order parameter, Q, at the crystal-nematic transition point was scaled up to unity. According to the original Bragg-Williams approximation, the complete ordered state (Q=

1) is occurred at the absolute temperature of zero. But in the case of p-azoxyanisole system, it is reasonable that the order parameter Q, which considers only the orientational order, is scaled up to unity at the crystal-nematic transition point because the orientational order is broken down gradually for the first time only after the crystal-nematic phase transition is occurred.

 $U_0$  in Eqs. (5) and (12), which is necessary to calculate the absolute thermodynamic properties such as the Helmholtz free energy, internal energy and enthalpy, was determined as follows. The entropy, vapor pressure and molar volume can be obtained without the  $U_0$  value [cf. Eq. (11)]. Thus using the relation of  $T\Delta S = \Delta H$  all the input values except  $E_N$  in Eq. (13),

$$\Delta H(N \rightarrow I) = (E_I + P_I V_I) - (E_N + P_N V_N),$$
(13)

can be calculated without  $U_0$ . The reasons are as follows: the quantities  $E_I$ ,  $P_I$  and  $V_I$  for the isotropic phase can be calculated from Eq. (7), and  $P_N$  and  $V_N$  for the nematic phase also can be directly obtained from Eq. (2) because Eq. (12) is not a function of volume. Equation (13) is reduced to Eq. (14) using  $E_N = E_{\text{bgd}} + E_{\text{conf}}$  (cf. Fig. 1) and Eq. (5) with Q = 0.

$$E_{\text{conf}} = \frac{R}{2k} U_0 = E_I + P_I V_I - E_{\text{bgd}}$$
$$-P_N V_N - \Delta H(N \to I)$$
(14)

Considerations of the Results. Fig. 2 shows that a little change in the slope of the line (pressure-temperature relation) occurs at the nematic-isotropic transition point. This may be due to the difference between the heat of vaporization of the nematic phase and that of the isotropic phase.

The calculated values of the thermodynamic properties are in good agreement with experimental results. The discrepancy between theoretical and experimental values is most significant in the vicinity of the anisotropic-isotropic phase transition point. The discrepancies in the molar volume, expansion coefficient and compressibility may arise from the fact that the Bragg-Williams approximation has no volume term which is a function of the order parameter. So the more advanced theory of order-disorder, which considers explicitly the volume term in the order parametric function, is necessary to obtain better predictions at the phase transition. But the discrepancy in the heat capacity may be improved by the application of the other theory of orderdisorder such as Kirkwood's method35 although this method does not also contain the volume term.

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